Dispersion Trading

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Dispersion Trading

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Dispersion Trading

Abstract

The main goal of this thesis is to study the dispersion trading strategy applied for the Brazilian options equity market. Some studies for developed markets show that implied correlation tends to be priced higher than the realized correlation. In order to verify and capture this difference, dispersion trading strategy can be used to trade index volatility versus the volatility of its components.

Brazilian options market does not present enough liquidity to trade stock options in all index components. Given that, an index tracking approach is presented to create an index proxy that tries to replicate the index return with a small subset of components.

This thesis explores multi-period strategies and different compositions for the index proxy portfolio in order to verify if the strategy is consistent.

The results provide strong evidence that dispersion trading is feasible to be implemented, being profitable even with positive transaction costs.

Keywords: Dispersion Trading, Correlation, Volatility, Brazilian Options Market
Executive Summary

Dispersion Trading is a trading strategy used widely in developed markets. The main focus of this strategy is to trade the dispersion of an index versus all its components. This study presents a case of study for the Brazilian Market and shows how the dispersion trading can be implemented for the local market.

The strategy was analyzed from 2010 until mid-2015 using different scenarios and considering transaction costs.

The main result of this study is that it is feasible to be implemented and provides a significant return rate. It is consistent with the idea that there is a long dispersion risk premium available in the market and it can be captured with this strategy.

The returns obtained are robust. Even after considering transaction costs, this strategy is profitable and could be implemented in the Brazilian market.
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Chapter 1

Introduction

Since option pricing model publications started with Black & Scholes [1], volatility is getting more interest from academics and market researchers. Volatility is a key concept used widely in financial markets. It can be defined as a statistical measure of the returns dispersion for a particular security or index. This measure gives us an idea about how risky a security can be.

An important concept about volatility to have in mind is that it does not measure the direction of the price change. It measures the intensity of price change but not the direction of the movement.

When analyzing volatility for an index another important concept appears. It is correlation. In the Financial Markets, correlation between two assets can be defined as a statistical relationship between their returns. It is an important metric because it can give the idea of what happens with one asset when the other moves in a given direction.

Analyzing the volatility of an index corresponds to analyzing the volatility of each
component individually weighted by their index participation and incorporating the correlation between them. In this scenario, correlation plays a key role in obtaining the volatility of an index based on the volatility of its components.

A complementary concept for correlation is dispersion. Dispersion measures how clustered the returns are for a particular set of securities. If dispersion is high the returns for the securities can be very different. However, if dispersion is low all securities tends to have the same return.

In general correlation and dispersion are negatively correlated. When correlation is low, dispersion tends to be high and vice-versa. High correlation means returns are directionally the same, while low dispersion means the magnitude of returns tends to be the same. A low dispersion situation can be viewed as a cluster of returns that are almost all at the same level.

Historically in the financial markets, index volatility tends to be priced higher compared to volatility of its components [2]. This can be explained because many traders tend to buy protection for the portfolio based on an index instead of a single stock. In this scenario it is possible to verify that the implied correlation is priced higher than the realized correlation.

Given that index volatility tends to be higher than the volatility of the basket with all index components and correlations between them, a trading strategy can be used to take advantage of this fact. This strategy is called Dispersion Trading. The strategy basically consists in shorting index volatility and buying volatility of its components. The opposite scenario can also be traded (buying index volatility and shorting volatility of all its components) but, in practice, this scenario is not common.
The scenario described above lead us to an interesting concept. If, in general, a strategy that is shorting index volatility and longing its components is profitable, then it can be concluded that there is a short correlation risk premium that could be capture from the market. This short correlation risk premium can also be interpreted as a long dispersion risk premium.

In order to implement the strategy, the options market can be used to buy or sell volatility. An important aspect that makes this strategy popular in developed markets is the fact that they have a highly liquid options market. However, the Brazilian options market does not have enough liquidity for all components that belong to the index. Only a few stocks are liquid for multiple strikes and maturity dates on the Brazil Exchange. To try to minimize this fact and be able to create a feasible implementation of dispersion trading, an index tracking approach will be used to replicate the index return payoff using a small subset of stocks.

Dispersion Trading is widely used into Developed Markets due the high liquidity available for options market and other derivatives. There are some studies presenting evidences that it is possible to capture a long dispersion premium available in the market for US [4, 5] and Europe [3]. These analyses indicates that the use of a dispersion trading strategy provides additional value to an investor. There is also an analysis of South Africa market [6] presenting good results for dispersion trading.

The focus of this study consists of analyzing the dispersion trading strategy for the Brazil Market and understanding the challenges that are necessary to implement this and analyze the output results that can be achieved from implementing this strategy.
This thesis is organized as follows: Chapter 2 introduces the concepts of return, volatility, correlation and dispersion. Additionally, pricing model and risk sensitivity for options are presented. Chapter 3 gives an introduction about the Brazilian options market and its particularities. Chapter 4 presents dispersion trading and how to implement this. Chapter 5 shows all data used for this study and how it was manipulated. Finally, Chapter 6 presents the results and analysis discussion and Chapter 7 concludes.
Chapter 2

Methodology

This section introduces the concepts that will be used for this study. Return, volatility, correlation and dispersion are discussed in the sections below. There is also an introduction for options pricing and sensitivity analysis.

2.1 Return

Financial data as stock price does not present stationary characteristics when analyzing its time series for a given period of time. Given that, it is important to transform the price time series into another time series that presents stationary characteristics [7]. After this process, it is possible to apply statistical inference. The most common way to analyze a price time series is using the first difference. This price difference is known as return and can be defined as the equation below:

\[ r = \frac{S_t - S_{t-1}}{S_{t-1}} \]  \hspace{1cm} (2.1)
where $S_t$ is the price for the security for day $t$ and $S_{t-1}$ is the price for the security for day $t - 1$.

Another way to calculate the return is using the log return concept. The log return approximation framework presents many good characteristics that makes this concept useful and massively applied in the financial industry [8]. The log return is defined by the equation below:

$$r = \ln \left( \frac{S_t}{S_{t-1}} \right)$$ (2.2)

An important point to notice about log return is that it is close to the traditional return for small returns. By this reason, it is normally used for daily returns. The equation below shows this using a mathematical definition.

$$\ln(1 + r) \approx r, r \approx 0$$ (2.3)

The return of an index can be defined as a return of a basket. It is basically the weighted sum of each component return that belongs to the index. The equation below presents the return for the index.

$$r_I = \sum_{i=1}^{N} w_i r_i$$ (2.4)

where $w_i$ is the weight of component $i$, $r_i$ is the return of component $i$ and $N$ is the number of components.
2.2 Volatility

Volatility is widely used in the financial market as a measure of uncertainty of a stock return. It provides important information about the uncertainty of asset return movements and it also can be interpreted as a measure to determine the risk of a particular security. In order to price an option, one of the main factors that is considered for this is the probability of an option be exercised in the future. If the volatility is high, there is high uncertainty about the price level level of a security in the future. This impacts the probability of an option be exercised or not. For this purpose, volatility plays a crucial role in the option pricing models.

Volatility can defined as the standard deviation of returns. It is equivalent to as defining the volatility equal to the square root of the variance of the returns. An unbiased estimator for standard deviation of returns is presented in Equation 2.5.

\[
\hat{\sigma} = \sqrt{\frac{\sum_{t=1}^{T} (r_t - \bar{r})^2}{T - 1}}
\]  

(2.5)

where \( r_t \) is the return for the day \( t \), \( \bar{r} \) is the average return and \( T \) is the number of periods considered.

This estimation method gives equal weights for all returns. There are also different methods to define volatility that can give more weight to recent events rather than to past events [9]. For this study only this traditional methodology of using equal weights for past dates will be used.
2.2.1 Realized Volatility

Realized volatility is an estimation of volatility based on historical returns. As presented above, an unbiased estimator for standard deviation of historical daily returns can be used. An important point to note is that volatility is normally annualized while returns are daily. The equation below presents the estimator with the annualized conversion:

$$
\hat{\sigma}_{Realized} = \sqrt{\frac{252}{T-1} \sum_{t=1}^{T} (r_t - \bar{r})^2}
$$  \hspace{1cm} (2.6)

2.2.2 Implied Volatility

Implied Volatility is defined as the volatility that can be extracted directly from the prices observed in the market. Unlike the price of an option, volatility is not a parameter that is easily observed and needs some extraction process to retrieve this. In order to obtain the volatility, an option pricing model needs to be used. The most famous model is the Black & Scholes model and it will be discussed in more detail in Section 2.5.1. The extraction process consists in determining the volatility used as input into the Black & Scholes formula that will result in the option price observed in the market. As it can be noted, the implied volatility is totally dependent of the model used to extract it.

Another approach to observe index implied volatility is using a volatility index. One example is VIX, it is a volatility index for the US market created by CBOE (Chicago Board Option Exchange) [10, 11]. It measures 30-day expected volatility of the S&P 500 Index. For Brazil, there is a recent study that constructed a volatility
index for IBovespa called "IVol-BR" [12]. It is based on daily market prices of options over Ibovespa.

These volatility indexes measure the market’s expectation of volatility implicit in the prices of options. They are also viewed as leading barometers of investor sentiment and market volatility relating to listed options.

\section*{2.3 Correlation}

Correlation measures the strength and the direction of a linear relationship between two variables. It is an important concept when studying volatility of a basket.

It can be defined as follows:

\[ \rho_{i,j} = \frac{\text{cov}(r_i, r_j)}{\sigma_i \sigma_j} \] (2.7)

where \( \text{cov}(r_i, r_j) \) is the covariance between returns \( r_i \) and \( r_j \), \( \sigma_i \) is the volatility for security \( i \) and \( \sigma_j \) is the volatility for security \( j \).

The volatility of a basket can be defined as the square root of the variance of a weighted sum and respective correlations [13]. The equation below presents the definition for basket variance:

\[ \sigma^2_{\text{Basket}} = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + 2 \sum_{i=1}^{N} \sum_{j>i}^{N} w_i w_j \sigma_i \sigma_j \rho_{ij} \] (2.8)

where \( \sigma_i \) is the volatility for \( i \)-th component, \( w_i \) is the weight for \( i \)-th component and \( \rho_{ij} \) is the correlation between components \( i \) and \( j \).

Note that correlation plays an important role as it defines the interaction between two different stocks. Based on Equation 2.8 it is possible to derive a measure of
average correlation. This measure represents the level of correlation between the index and all its components. It is defined in the equation below:

\[
\bar{\rho} = \frac{\sigma^2_{\text{Index}} - \sum_{i=1}^{N} w_i^2 \sigma_i^2}{2 \sum_{i=1}^{N} \sum_{j>i} w_i w_j \sigma_i \sigma_j}
\]  

(2.9)

Using this formula for average correlation, implied and realized correlation can be calculated using observed market data and historical data, respectively.

### 2.4 Dispersion

Correlation is a good metric to show the directional movement for returns but it does not indicate the magnitude of them. Dispersion arises from this context. It tries to measure how disperse are the returns of components compared to the return of the index.

If we consider a portfolio with correlation equal to 1, the volatility of this portfolio can be defined as the average volatility weighted by each component weight. The definition is presented in the equation below:

\[
\sigma^2_{\text{Basket}} = \sum_{i=1}^{n} w_i \sigma_i^2
\]  

(2.10)

This totally correlated portfolio provides us an important barrier for the maximum volatility that can be obtained for this. The dispersion metric can be defined as this maximum volatility minus the index volatility. This measure give us the idea of how disperse our portfolio is compared to the index. The equation below presents the definition for dispersion:
As discussed for correlation, the realized dispersion can be calculated using the historical data and implied dispersion can be calculated using the volatility obtained from the market observed data.

2.5 Option Pricing Models

Option pricing theory has evolved since the Black & Scholes article in 1973 [1] that presented a satisfactory equilibrium option pricing model where they derived a partial differential equation, well known as the Black-Scholes equation. Also in the same year, Robert Merton extended their model introducing mathematical understanding of Black & Scholes equation [14]. The union of Black & Scholes and Merton studies represents a change in the industry about option pricing and created the Black & Scholes options pricing model.

There are also other options pricing methods that appears in order to calculate option price using different approaches [15, 16]. These methods try to enhance some restriction in the Black & Scholes as constant volatility and exercise only at the maturity date.

2.5.1 Black & Scholes Option Pricing

Black & Scholes model assumes that underlying stock price follows a geometric brownian motion. It means that the model assumes a constant drift and volatility
for stock price movements. The Geometric Brownian Motion formula is defined as below:

\[
\frac{dS_t}{S_t} = \mu dt + \sigma dW_t
\]  

(2.12)

where \(\mu\) is the drift, \(\sigma\) is the volatility and \(dW_t\) is the Wiener process.

Black-Scholes formula can be derived based on the geometric brownian motion for stock price movement and other assumptions as: no arbitrage, option payoff replication using a continuous delta hedging approach and option exercise only at the maturity date.

Equation 2.13 represents the Black & Scholes formula for call options and Equation 2.14 represents the formula for put options.

\[
C(S, t) = \Phi(d_1)S - \Phi(d_2)Ke^{-r(T-t)}
\]  

(2.13)

\[
P(S, t) = \Phi(-d_2)Ke^{-r(T-t)} - \Phi(-d_1)S
\]  

(2.14)

\[
d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma \sqrt{T-t}},
\]

\[
d_2 = d_1 - \sigma \sqrt{T-t}.
\]

where \(\Phi\) is the cumulative distribution function of the standard normal distribution, \(S\) is the spot price, \(K\) is the strike price, \(T-t\) is the time to maturity, \(r\) is the risk free rate and \(\sigma\) is the volatility.
2.6 Risk Sensitivity

As options depend on different factors such as underlying price and volatility, it is important to have some tools available to analyze the scenarios that can happen given some shocks into these factors.

These risk sensitivities could be derived using the partial derivative of option pricing model for each particular factor.

The most important sensitivities that will be studied are delta ($\Delta$), gamma ($\Gamma$) and vega ($\nu$). Delta is the partial derivative of option price with respect to the underlying security price. Gamma is the second partial derivative of option price with respect to the underlying security price. And finally, Vega is the partial derivative of option price with respect to the underlying security volatility.

In the sections below each sensitivity will be discussed in more details.

2.6.1 Delta

Delta is defined as the first derivative of option price with respect to the underlying security price. It measures the rate of change of the option price when there is a change in the underlying security price. The equation below shows delta definition:

$$\Delta = \frac{\partial V}{\partial S} \quad (2.15)$$

where V is the option value and S is the underlying price.

It is a broad measure used when working in the options market. Another way to think about delta is that it represents the quantity of the underlying security necessary to build a position that replicates the price variation of the option given a
small change in the underlying security price. As option price does not follow a linear formula, it is necessary to recalculate the delta for each price change or any other factor change that could impact the option price.

Delta hedging is the process to protect the portfolio from the price variation of the underlying security. In order to have a perfect hedge for the portfolio a continuous delta hedging needs to be considered to recalculate the delta and rebalance the portfolio for each change in any factor that could impact this. If continuous delta hedging was possible to implement its cost would be high due the high number of transactions. In general, delta hedging is made daily, normally at the end of the day.

The Black & Scholes formula for delta are presented below:

\[ \Delta_{\text{call}} = \Phi(d_1) \]  
\[ \Delta_{\text{put}} = -\Phi(-d_1) \]  

2.6.2 Gamma

Gamma is defined as the second derivative of option price with respect to the underlying security price. It measures the rate of change of the delta when there is a change in the underlying security price. Gamma is defined by Equation 2.18 and Equation 2.19 presents the Black & Scholes formula for gamma.

\[ \Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 V}{\partial S^2} \]  
\[ \Gamma = \frac{\phi(d_1)}{S \sigma \sqrt{T-t}} \]
where $\phi$ is the standard normal probability density function.

### 2.6.3 Vega

Vega is defined as the sensitivity of option price given a change in the volatility. This represents the amount of money that will impact the option price given a change in the volatility. It can be defined as the equation below:

$$\nu = \frac{\partial V}{\partial \sigma} \quad (2.20)$$

Vega can be obtained from two different equations using Black & Scholes formula. The equations are presented below:

$$\nu = S\phi(d_1)\sqrt{T - t} \quad (2.21)$$

$$\nu = Ke^{-r(T-t)}\phi(d_2)\sqrt{T - t} \quad (2.22)$$
Chapter 3

Brazilian Market Particularities

This chapter presents the concepts that are particular for the Brazilian options market. The first section presents an introduction for the Brazilian market. In the second section the options market liquidity is discussed and finally the last section contains details about dividend adjustments made for options contracts.

3.1 Options Market

In Brazil there is only one exchange with mandate to trade Stocks, Options and Futures. The actual BM&FBovespa was born in 2008 from the merger between Bovespa (Sao Paulo Exchange for Stocks and Options) and BM&F (Mercantile and Futures Exchange for Futures and Options Futures).

Today the main products available at BM&FBovespa are stock, stock options, index options, index futures, currency futures, options futures and others. For this particular study we will concentrate on stock options, index options and stock prod-
Regarding the options market, exchange has option that follow two different styles: American and European. American options can be exercised any time until maturity date. However, European options can only be exercised at the maturity date.

Stock options are available for American and European styles. Expiration date is always the third Monday of maturity month, or the subsequent business day, and it is available for every month.

Index options are only available as European style. Expiration date is always the closest Wednesday of 15th day of the maturity month or the next business day. It is available for all months.

3.2 Market Liquidity

In 2011, Bovespa Exchange started a market maker program that introduced more liquidity into the market by having brokers responsible for proving liquidity for some symbols. Unfortunately, even after the market maker program implemented by the Exchange, there are still only a few symbols that are liquid and can be used to implement the strategy. As of Sep, 2015, Bovespa Index had 64 stocks in its composition but only around 15 components are liquid in the option market and could be used.

Liquidity is a key point to implement Dispersion Trading. Ideally all index constituents volatility should be used to replicate the index volatility. But in some circumstances like this, there are another approaches that could be used in order to implement the dispersion trading strategy. These approaches will be discussed later.
3.3 Dividend Adjustment

The options market in Brazil has a particular concept of adjusted options positions. As described in the exchange manual [17] there are two type of options in Brazil subjects to corporate actions: adjusted options and non-adjusted options.

As stated in the document: "On the options market, rights and yield, including dividends, bonuses, warrants and others conferred on underlying securities on the cash market will be treated differently depending on whether the options are adjusted or non-adjusted".

Adjusted options have the strike adjusted when there is a corporate action that impacted the price of its underlying. Basically the amount paid as dividend by a stock is deducted from the strike price. This type of option is common for stock options.

Non-adjusted options are not adjusted when there is a dividend payment or any other corporate action. These options are common for index options. As Brazil Index follows the total return concept, it is not necessary to adjust by dividends. Total return index means that all dividends and others corporate actions are included in the final index points. As stated in the exchange document for index methodology [18]: "The adjustment is made considering that the investor sold the stocks at the closing price of the last trading day prior to the benefit distribution and used the resources to buy the same shares without the benefit (i.e., at the "ex-theoretical" price)".
Chapter 4

Dispersion Trading

Dispersion Trading is a strategy that trades the dispersion of index constituents against the index. In a traditional index arbitrage [19, 20] or statistical arbitrages [21], the core arbitrage concept is return. These strategies try to replicate the return of an index using different approach in order to obtain a gain with the arbitrage. For Dispersion Trading the idea is to trade the dispersion. It is not a true arbitrage as, for example an index replication with future contracts and stocks. It is a long and short strategy that tries to capture the correlation risk premium. As the focus of this strategy is not to trade return, derivatives need to be used in order to be able to trade correlation (or dispersion). A straightforward way to trade correlation is trading volatility of an index against the volatility of a basket. One possible market that allows us to trade volatility is the options market. Options Market can also be interpreted as an insurance market where there are agents selling insurance for others. These insurances can be interpreted as the right to buy or sell a financial asset in the future. As many portfolio managers tend to buy index protection for their portfolio,
by the concept of supply and demand it is possible to verify that index options are priced higher than stock options [2].

Another way to analyze the reason why index options tend to be priced higher is based on the fact that index option sellers are assuming a risk of correlation when writing an option. The actual correlation that was used to calculate the index volatility is not the same correlation that will realize in the future. As correlation risk is not simple to hedge using traditional products available in the market, index option sellers needs to include a premium to carry this uncertainty.

This section will cover the concepts related to dispersion trading and also explain how index tracking will be used to replicate the index in a market with restricted liquidity.

4.1 Strategy

What does trading dispersion really mean? What exactly is this strategy trying to achieve? In order to answer these questions let’s analyze two scenarios, one for high dispersion and another with low dispersion. These scenarios are illustrated in the Table 4.1 and Figure 4.1.
Analyzing the first scenario for high dispersion, it can be noted that even with a different magnitude and direction for each stock return, the return of the index is not affected. Having big return movements will cause the volatility to increase in the future. This scenario can also be viewed as a low correlation as the direction of the returns are not the same. In this scenario if we buy volatility of index components and sell the volatility of the index we can make money exploring this fact.

From the other side, if the direction and magnitude of returns are the same it is a scenario where there is a low dispersion and consequently high correlation.
This basically illustrates the scenarios that we want to explore in order to enter into a trade that can capture this risk premium embedded.

As already mentioned by the difference for volatility pricing, the classical strategy to implement dispersion trading is to sell index volatility and buy volatility in all its constituents.

In order to normalize both sides of the strategy the concept of notional equivalency can be applied. This means that for each index option contract an equivalent basket of options will be traded with the same size. The equation below represents this:

\[ n_i = n_I \times \frac{w_i K_I}{K_i} \]  

(4.1)

where \( n_i \) represents the numbers of contract of asset \( i \) that needs to be traded, \( n_I \) is the number of contracts for the index, \( K_I \) is the index strike price and finally \( K_i \) is the strike price for \( i \)-th component.

### 4.2 Implementation

Dispersion Trading can be implemented in different ways. The only necessary artifact to build the strategy is the use of a financial product where it is possible to trade volatility. One possible implementation is to use a vanilla call option with delta hedging. This implementation is good from the perspective of simplicity but the problem is that the cost of delta hedge is high.

Option straddle is another strategy to buy or sell volatility in the asset underlying. The strategy consists in buying a call and a put option for the same strike and maturity for a given asset. The advantage of using a straddle instead of just buying
a simple call option is because the cost of delta hedging of the straddle is small. As straddle consists of a position in a call and a put option, the positive delta from call option is reduced by the negative delta from put option. Given this, the net delta position from the straddle is smaller than a delta position of a simple call option. Using straddles to enter into a dispersion trade consists in being short in index straddle and long in all its components straddles.

Just as a note, in developed markets variance swaps are one of the main instruments used to implement dispersion trading [22]. In Brazil there is still no liquidity in the stock options market for this product. However, there are some studies about variance swaps pricing models for foreign exchange rate market [23, 24].

4.3 PnL Measurement

As dispersion trading consists of a long and a short position, it is almost a self-funded strategy. The amount of money necessary to enter into a dispersion strategy is relatively small.

The premium paid to be long volatility is almost the same as the premium received to be short volatility. Given that, a return measure needs to be defined in order to analyze each strategy with the same parameters.

The profit and loss (PnL) computed for the strategy consists in summing up all payments (paid or received) for all straddles positions and delta hedging. This gives the amount of money earned (lost) per strategy.

As our strategy is based on index option contracts on one side and its components on another side, the index size is a good value to be used as a normalization de-
The equation below presents the rate of return for the dispersion trading strategy:

\[ r = \frac{PnL}{K_I} \]  

(4.2)

where \( PnL \) is the sum of all cashflow and \( K_I \) is the index strike price.

As the PnL was computed for a given period of time, it is important to annualize this value to make this strategy comparable with others. The formula used to annualize the return is described below:

\[ r_{\text{Annualized}} = (1 + r)^{\frac{252}{\text{days}}} - 1 \]  

(4.3)

This provides us a more normalized way to represent the return of the strategy and be able to compare with others.

There are also another techniques to measure the PnL of a long and short strategy. One example could be considering a given cash amount that is used to build the portfolio and perform delta hedge. After portfolio is built the remaining cash is accrued at the risk free rate. The total return obtained of the strategy can be compared with the initial cash amount accrued only by the risk free rate.

### 4.4 Correlation Risk Premium

The Correlation Risk Premium can be interpreted as the difference of the implied volatility of the index vs the volatilities of each component. As discussed, the volatility of the index tends to be priced higher. One of the reason is the correlation risk that option writers need to carry. Correlation is not simple to trade in the market and it
is difficult for the option writer to protect from correlation changes. There are some studies showing evidences of correlation risk premium [25].

For this reason, a correlation risk premium is embedded into index options pricing in order to give some extra incentive for the option writer to enter into a trade. As there is a negative correlation between correlation and dispersion, short correlation risk premium can be also interpreted as long dispersion risk premium.

4.5 Index Tracking

Given the limitation in Brazilian options market that not all index constituents have enough liquidity to be traded an index tracking approach needs to be used. There are more advanced studies that uses genetic algorithms [26] and Principal Component Analysis (PCA) [5] in order to select the optimum portfolio. For this study, an optimization method was used to calculate the optimal weights that best replicates index past returns for a given period of time. The approach used consists in minimize the square error between index return and the proxy index portfolio return. The error is represented in the equation below:

$$
\epsilon = \left( r_I - \sum_{i=1}^{N} w_i r_i \right)^2
$$

where $r_I$ represent the index return, $r_i$ is the return for $i$-th component, $w_i$ is the weight of the $i$-th component and $N$ is the number of components used.

The equation below illustrates the formula used to minimize $\epsilon$ for multiple days in a given period of time $T$. 

$$
\epsilon = \left( r_I - \sum_{i=1}^{N} w_i r_i \right)^2
$$
\[
\min_w \sum_{t=1}^{T} (r_{I,t} - \sum_{i=1}^{N} w_i r_{i,t})^2 \\
\text{s.t.} \quad \sum_{i=1}^{N} w_i = 1 \quad w_i > 0, \forall i \in N.
\] (4.5)

It is important to notice that there are two restrictions for the minimization. The first is that the sum of all weights needs to be 1 and the second one is that the weights are strictly positive. After running this minimization, the portfolio can be constructed with the optimal weights for each component.
Chapter 5

Data Analysis

All data used for this study was extracted from BM&F Bovespa and Economatica. From BM&F Bovespa stock, index and options closing price, options specifications and interest rate term structures were extracted. From Economatica closing price adjusted by dividends for each stock was extracted. This closing price adjusted by dividends is important to obtain the daily return eliminating the variation that could be caused by dividends.

Prices and option specifications were obtained from historical quotes from Bovespa. The risk free rate was obtained from BM&F using the benchmark rates ”ID x Fixed Rate”. It provides a daily term structure curve for Brazil interbank deposit certificate rate (CDI).

The period studied was between January, 2010 and June, 2015.
5.1 Return Extraction

Historical returns are an important piece of data to build the dispersion trading position. As discussed before they are important to determine realized volatility and also the index components weights using the index tracking approach. For this study, Economatica source was used because it provides a price series already adjusted by dividends and corporate actions. This adjustment is important to make straightforward the return calculation across different periods of time. As discussed in Section 2.1, returns were calculated using the log return methodology.

5.2 Implied Volatility Extraction

To obtain the option price using the Black & Scholes formula there are six parameters necessary as inputs into the formula: stock price, option strike, option maturity, volatility, option type and risk free rate. As defined in Section 2.2.2, the implied volatility is the volatility used as input for the pricing model that will result in the option price observed in the market. As all other variables and the option pricing are known, the only variable that needs to be obtained is the implied volatility. For this study, Black, Scholes & Merton model will be adopted to extract the implied volatility from option price. This model considers the options as European style.

However, in the Brazilian options market there are European and American styles for stock options. Naturally, there are some differences between option pricing for each type. For call options, the price difference only happens when there is a dividend payment [27]. As discussed before, in the Brazilian options market the option strike is
adjusted when there is a dividend. This makes the use of Black & Scholes consistent for both American and European call options.

On the other hand, for put options it is not trivial as call options to determine the price of an American option versus an European option. There are more scenarios rather than dividend payment where it is beneficial to early exercise a position. In general, the strategy consists in early exercise the put option in order to receive money and earn interest on it. These scenarios basically occur when the put option is deep in the money [28].

The database with option specifications used for this study did not include the style for stock options. For index options it is assumed by definition to be always European but for stock options it is not possible to know. In order to simplify this, deep in the money put options where not considered as part of this study and all options were calculated using Black & Scholes formula, i.e. considering them as European options.

Another issue found during the implied volatility calculation was that the price data for stocks and options available at the exchange is only the closing price. In this situation, when a non liquid option contract is negotiated at the begin of the day and there are no more trades for that given day, the close price will be the price traded at the beginning of the day. This can lead us to inconsistent results because the spot price that is inputted in the pricing model will be the closing price and not the current price at the moment that the trade was executed. This can cause some distortions in the implied volatility calculation.

As the stocks used in this study belong to the index, the liquidity for stock market
is very high and the last price is the one that was traded in the closing auction, but for options this is not a case in all scenarios. Normally OTM (out of the money) and ATM (at the money) options have more liquidity but there are scenarios where just few trades are executed per day. However, as this was the best data available for this study, stock closing price was used as spot price for implied volatility calculation and when constructing the volatility surface a special approach was adopted in order to minimize this effect. This approach will be discussed in the next section.

5.3 Volatility Surface Calculation

When observing option market it is possible to see that the level of volatility of two different strikes for the same expiration date is not the same. This behavior is called volatility smile [29, 16]. There are several theories trying to explain the existence of volatility smile: expectation of changes in volatility over time, support/resistance levels at various strike, crash protection and others are some possible explanations.

Volatility surface is a broader concept that combines all volatilities smiles by their expiration date in a surface. This is an useful resource used to obtain any particular volatility just passing two parameters: moneyness and expiration date.

Moneyness is a measure to normalize the strike level. It is defined as the equation below:

\[
m = \frac{K}{S}
\]

(5.1)

where \( K \) is the option strike and \( S \) is the underlying price.

Using moneyness it is possible to compare strike levels across multiple periods. And also it is simple to determine if an option is ATM (at the money), ITM (in the
money) or OTM (out of the money). For ATM options, which will be the focus of this study, moneyness is equal to 1. Using moneyness instead of strike level in the regression process to calculate the volatility smile makes the curve more consistent when comparing across different periods of time.

The regression process to obtain the volatility smile consists in getting all implied volatilities for the same expiration date and apply a polynomial regression to obtain the curve. There are other studies [30, 31] that suggest different approaches rather than polynomial regression to obtain the volatility smile. These methods are more consistent in terms of guarantee that there is no arbitrage rather than a simple polynomial regression. On the other side, polynomial regression is simple to calculate and produces an acceptable result. Then for this study polynomial regression will be used.

In order to minimize the effect that was discussed in the previous section regarding the fact that closing price for options does not reflect the price that was traded during the closing call at the exchange, an approach to use weighted polynomial regression was adopted [32]. Using the concept of weights during the regression process gives more relevance for options that are more liquid and consequently should make the volatility surface calculation using closing prices more accurate. There are two potential parameters to be used as weights: financial volume and number of trades. Financial volume reflects the sum of all traded amount for a given option in one day. This is a good measure to represent volume but it fails to capture liquidity of options. Even though the financial volume contains significant information about that particular option it is not a good metric for weight because we are using closing
data for spot and option prices. If for example, there is a large trade between two parties at the beginning of the day and no more trades were executed for this option for the rest of the day, the trade volume will be high but it is not reflecting that the option is liquid. From the other side, if the number of trades is used, it gives a better estimation about which option have enough liquidity and have a lower probability of having a big discrepancy between the option last trade price and closing price.

With this framework it is possible to apply the curve fitting process for each expiration date of all options. The result will be a polynomial curve that best fits the volatility points. For this study a 3rd degree polynomial was used.

After calculating all polynomial curves for each particular expiration we need to interpolate the curves to obtain the volatility surface. The method chosen to interpolate between the expiration dates was a simple linear regression. This provides a reasonable result for the purposes of this study.

In terms of consistency for the volatility surface, as the strategy is based for ATM options, the limit range for the moneyness in the volatility smile will be determined as 95% for the left side and 105% for the right side. This guarantees a volatility smile more consistent because any strike that is not inside this range will be considered as the edge value. This also makes the daily delta hedge more consistent.

After completing this step, we are able to price any option with any strike and any expiration date. This is important to price all options that were used in this study.
5.4 Implementation

In order to implement and validate dispersion trading a software was developed to perform each part of the processing as: data import from external sources into a database, implied volatility extraction, volatility fitting curve process, dispersion trading strategy evaluation and analysis/report extraction. The software was written in Java with libraries: JQuantLib [33] and Apache Math. To store all the preliminary and final data a MySQL database was used.
Chapter 6

Analysis and Results

To validate the results for Brazilian Market, the dispersion trading strategy was implemented using straddles. The strategy consists of entering into a position and holding it until maturity. The strategy is assumed to be always long dispersion, i.e. short index volatility and long volatility of the most liquid index components.

The index components chosen were based on the most liquid options of the last five days before the strategy starts. These components are then used to replicate the index based on the index tracking approach described in Section 4.5 for a period of 1 month. This replication process will result in an index proxy portfolio with respective weights for each component that will be used in this strategy.

The next step to build the dispersion trading strategy consists in constructing a portfolio of straddles for the same maturity. An important point to highlight regarding the Brazilian market is that the expiration date for stock and index options are different. Normally they are close but are not the same, as discussed in section 3.1. To build this strategy it will be assumed that both stock and index options expire
at the same date for simplicity. For example, in a real scenario, it is possible to use stock option expiration date and consider an OTC (over the counter) index option that expires at the same date.

The portfolio consists of short positions of ATM index straddles and long positions of ATM straddles of its components with their respective weights for the index proxy portfolio. To implement this portfolio it was assumed that the number of contracts for index straddles will be always 1 and the number of contracts for each component will be calculated using the Formula 4.1. After the portfolio of straddles is built the next step is to perform the delta hedging for each individual component and also for the index in order to buy or sell each component to zero the total delta value of the portfolio. This concludes the first day of the strategy.

Until the maturity date, the strategy is evaluated daily using the volatility surface extracted from market closing prices to perform mark to market and delta hedging process. In case of any stock pays dividend during the life of the strategy, the stock straddle will be rebalanced to keep the same weight that it had before the dividend payment.

This daily evaluation is important because it will give us the amount of money that was invested or received by the strategy daily. At the end of the period, the sum of all these values will result in the PnL for the strategy. This PnL is then converted in a rate of return annualized as described in the Section 4.3 to be able to compare different periods of time.

In order to perform a reasonable validation for this strategy 3 different periods of time with 3 different subset of components were chosen. The periods used were 1, 2
and 3 months to maturity and for the components were chosen the top 5, 10 and 15 most liquid stock options to compose the index proxy.

From 2010 to 2014, 12 portfolios per year (one for each month) were created and evaluated to achieve the average rate. For 2015 only 6 months were included.

The Figure 6.1 illustrates the difference between the implied volatility of the index against the implied volatility of all proxy index components. Unless it is explicitly denoted the following figures will always refer to the strategy with 10 components and 1 month to maturity. This figure also illustrates the gap between the index implied ATM volatility and the weighted average implied ATM volatility of its constituents. Basically it shows the difference in the volatility if the correlation between each component was zero. From these two graphs we can notice that correlation plays an important role when defining the volatility of a portfolio.

The table presented in Figure 6.2 shows the results for each strategy compared to CDI (Interbank deposit certificate rate) and IBOV (Brazilian stock index). This table contains the results for the whole period analyzed. The first two lines present the annualized return and volatility. The third line is the information ratio that is a metric of return divided by the volatility. Max Drawdown presented in the fourth line is the maximum consecutive loss that happened during the strategy life. For the rest of table it is presented the best and worst month and also the percentage of months with positive and negative returns.

This table also presents a color schema. Green cells represent good indicators and red cells are the bad indicators. Then a gradient between green and red was used to determine the colors of all cells based on its relevance.
Chapter 6: Analysis and Results

(a) Implied Volatility

(b) Implied Volatility - Zero correlation proxy index

Figure 6.1: The graph on the left illustrates the difference between the implied volatility of the index vs proxy index created to replicate the index. On the right, the graph illustrates the gap that is related to the correlation between the volatilities of each index proxy component.

The graph presented in Figure 6.2 illustrates the accumulated return for each strategy.

It is important to notice that dispersion trading strategies are self-funded when compared to CDI. Given that, the results obtained are robust because it is possible to have a return compared to CDI without committing all resources. Comparing this with IBOV, it is possible to see that the strategy is far less volatile given its long and short nature.

Still analyzing the results table, it is possible to note that short term strategies have a better annualized return but consequently a higher volatility. 3 months strategies have a lower volatility but consequently the return is also lower. When analyzing the other parameters, the 2 months strategies with 5 and 10 components present good
indicators for information ratio and number of positive month returns.

One main question when analyzing dispersion trading is from where exactly the return is coming. This is an fundamental question and it is part of what was already discussed in the previous sessions. The PnL basically comes from the fact that the market prices correlations higher than they really are. The Figure 6.3 shows the difference between the implied correlation vs realized correlation. It is possible to see that, in general, the implied correlation is higher than the realized correlation. This is in line with the idea that index options are priced higher than its components options.

Still analyzing the Figure 6.3 it is possible to notice that at the end of 2011 there is one example of implied correlation that is greater than 100%. There are two feasible explanations for this. The first one is the fact that we are comparing the index against a proxy index that does not contain all components and it can cause some distortion for the correlation. On the other hand, it is possible to analyze this scenario as a true arbitrage. Analyzing Equation 2.9, if the implied volatility for the index is higher and/or the implied volatility of its components is priced lower, it is possible to have a scenario where the implied correlation is higher than 100%. Per definition as it is not possible to have a realized correlation higher than 100% this can figure as a true arbitrage scenario.

The Figure 6.4 presents a dispersion graph that illustrate this relation. The x-axis represents the strategy return annualized and the y-axis presents the difference between implied correlation and realized correlation. It is possible to note that strategy tends to be profitable when the realized correlation is lower than the implied
(a) Descriptive Results

(b) Accumulated return

Figure 6.2: This figure presents a table showing the descriptive results and the accumulated return graph for each strategy.
correlation.

Figure 6.3: This figure illustrates the difference between implied correlation vs realized correlation. It is important to note that normally the implied correlation is greater than realized. This is in line with the idea that there is a short correlation premium in the market.

Figure 6.4: This figure shows the difference between realized and implied correlation vs the strategy return. When the realized correlation is lower than the implied correlation the strategy tends to have a position rate of return.
The same analysis that was made for correlation can also be applied for dispersion. As discussed before, dispersion trading strategy normally is a short correlation position that means a long dispersion position. From this concept to be able to apply the same analysis, the strategy should have a low implied dispersion and a high realized dispersion. This is basically what was observed during the period that was analyzed for this study. The Figure 6.5 shows the difference between implied and realized dispersion. It is possible to note that implied dispersion tends, in general, to be lower than realized.

![Dispersion Graph](image)

Figure 6.5: This figure shows the difference between implied dispersion vs realized dispersion. It is possible to note the same relation that was observed for correlation in Figure 6.3 but in the opposite direction. The implied dispersion tends to be lower compared to realized dispersion.

Figure 6.6 illustrates the dispersion graph of the spread between implied and realized dispersion versus the return. As it was observed for correlation, the strategy is profitable when the implied dispersion is lower than realized dispersion.
Figure 6.6: This figure shows the difference between realized dispersion and implied dispersion vs the return obtained by the strategy. When realized dispersion is higher than implied dispersion the strategy tends to have a positive return.

The volatility spread between implied and realized volatility for the index and proxy index can be defined as the difference between implied and realized volatility for the index minus the difference between implied and realized volatility for the proxy index as defined in the equation below:

\[
\sigma_{\text{spread}} = \left( \sigma_{I_{\text{Implied}}} - \sigma_{I_{\text{Realized}}} \right) - \left( \sigma_{P{I}_{\text{Implied}}} - \sigma_{P{I}_{\text{Realized}}} \right)
\]  

(6.1)

This spread can be observed in Figure 6.7. For this spread be negative, the difference between implied and realized volatility for the index proxy needs to be greater than the difference between implied and realized volatility for the index. In this case, it is possible to observe that the return tends to be positive. This is in line with the concept of dispersion trading that was discussed above that there is a long dispersion premium available in the market. The realized volatility for index
components tends to overcome the realized index volatility relatively to the implied volatilities.

![Volatility Spread vs Return](image)

Figure 6.7: This figure shows the dispersion graph for the spread between implied volatility and realized volatility of index minus the spread for the proxy index against the strategy return.

The Figure 6.8 shows the total vega for the strategy over the time. It is interesting to note that vega is small when it is close to start and maturity dates. For the other days, it is possible to see that the strategy is not totally clean, in terms of vega, and volatility changes in index and stocks could impact the strategy profit.
Chapter 6: Analysis and Results

Figure 6.8: This figure presents the total vega for the strategy over the time. It was built considering the daily vega of 1 month strategy.

Until this point, all the analyses were made with the absence of transaction costs. After analyzing this strategy in terms of expected return we proceed to understanding if this strategy is feasible to be implemented in a real scenario. Given this, an analysis of transactions costs is important to understand what is the price paid to enter into a dispersion trading strategy for the Brazil market.

When discussing about transactions costs there are multiple type of costs that can be considered. The most obvious is the commission paid when trading a security. For the options market the commission is a percentage applied to the notional of the contract. Together with broker commission it is important to consider also the middle monies that are paid for the exchange and clearing house. All these fees together compose the cost necessary to enter in a option position. Another point to have in mind, when the option expire with a positive payoff, more cost needs to be considered because the trade executed when an option contract exercises also requires commissions. And finally, for stock options, as the exercise in the Brazilian options market is physical the cost to sell the stocks received in order to have only the cash position should also be included.
The second type of transaction cost that can be considered is the bid-offer spread. For the Brazilian options market, as the liquidity of the market is limited this can be an important cost that needs to be analyzed carefully.

The last transaction cost that could be accounted for this strategy is the delta hedge cost. Delta hedging process basically consists in rebalancing daily the portfolio to zero the delta of all open option positions. This involves a high number of trades and can lead to a costly process. When this strategy is being implemented in a large trading desk of banks or in a hedge fund, the cost of delta hedging can be shared with other strategies. In other words, for example if dispersion trading strategy requires to buy the stock A to zero a negative delta of a put option but there is another strategy of the trading desk that has positive delta for this stock A, the net delta can be considered and hedged just the difference. This will result in a smaller trade that will manage the cost in a more efficient manner. As this cost can be split for multiple trading desks it will not be considered for this analyze.

For this study, the exchange and clearing house costs considered were based on the Bovespa cost table: 9.5 bps for stock options, 6 bps for index options, 3.25 bps for index exercise and 2.5 bps for stocks exercise and standard trades. Commission fee assumed was 2 bps for all trades. And finally, for bid-offer spread it was assumed a range of 1 volatility for index options and 2 volatilities for stock options. This means, if the volatility obtained from the surface for a stock option was, for example, 20% then it will be considered as 21% to calculate the price to buy this option. For daily evaluation, to calculate the delta hedging and mark to market it was used the volatility obtained directly from surface without any volatility bump.
The results table considering transactions costs are presented in the Figure 6.9. This table shows that the results are outstanding even when considering transaction costs. The parameters used for the analysis are the same that were used before without the transaction costs. As expected the behavior is the same, short term strategies have higher return with higher volatility while 3 months to maturity strategies have lower return with lower volatility. Also looking for the other parameters the strategy with 5 and 10 components with 2 months to maturity continues to be the best choice.
Figure 6.9: This figure presents a table showing the descriptive results and the accumulated return graph for each strategy considering the transaction costs involved.
Chapter 7

Conclusions

Dispersion Trading is a trading strategy used widely in developed markets. This study presented a case of study for the Brazilian Market and the results are in line with the initial expectations. The strategy is feasible to be implemented and provides a significant return rate. It is consistent with the idea that there is a long dispersion risk premium available in the market and it can be captured with this strategy.

One important point to note about this strategy is that the capital demanded to enter in the strategy is small compared to the notional. It is an almost self funded strategy. As it is composed by a long and short position the premium paid to enter into a long position is almost offset by the premium received to be in a sell position. As the initial resource necessary to enter into the strategy is small, the return can be leveraged to produce more return without the necessity of allocating all capital for this strategy.

There are also some points that could be improved to make this strategy more stable and cleaner from the perspective of the sensitivities studied. For example,
to avoid vega interference the straddles portfolio could be rebalanced daily in order to minimize its impact. Another methods to obtain the index proxy portfolio could also be used in order to minimize the fact that not all components that belongs to index could be used. In the future, when new derivatives products as variance and volatility swaps have enough liquidity in Brazil, these also could be used to implement this strategy.

Finally, the returns obtained for the period from 2010 until mid 2015 are robust. Even after considering transaction costs, this strategy is profitable and could be implemented in the Brazilian market.
Bibliography


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